Real-Life Math

PROBABILITY

SECOND EDITION

WALCH PUBLISHING

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Appendix

How to Use This Series

The *Real-Life Math* series is a collection of activities designed to put math into the context of real-world settings. This series contains math appropriate for pre-algebra students all the way up to pre-calculus students. Problems can be used as reminders of old skills in new contexts, as an opportunity to show how a particular skill is used, or as an enrichment activity for stronger students. Because this is a collection of reproducible activities, you may make as many copies of each activity as you wish.

Please be aware that this collection does not and cannot replace teacher supervision. Although formulas are often given on the student page, this does not replace teacher instruction on the subjects to be covered. Teaching notes include extension suggestions, some of which may involve the use of outside experts. If it is not possible to get these presenters to come to your classroom, it may be desirable to have individual students contact them.

We have found a significant number of real-world settings for this collection, but it is not a complete list. Let your imagination go, and use your own experience or the experience of your students to create similar opportunities for contextual study.

Foreword

You've seen it happen many times—a player in a dice game claims she is "due" for doubles; strangers discover that they have a mutual acquaintance and think that this must be more than a chance meeting; a friend plays the lottery obsessively or enters online contests with a persistent dream of winning. All these behaviors reflect how people perceive probability in daily life. People who lack an accurate sense of probability are easily drawn in by false claims and pseudoscience, are vulnerable to get-rich-quick schemes, and exhibit many of the behaviors mentioned above.

The modeling and measurement of probabilities are fundamentals of mathematics that can be applied to the world around us. Every event, every measurement, every game, every accident, and even the nature of matter itself is understood through probabilistic models, yet few people have a good grasp of the nature of probability. Even students who have taken typical mathematics courses are unlikely to acquire the mathematical skills necessary to apply probabilistic models to real-world situations.

This book will help fill the gaps. This collection of activities will supplement general math, pre-algebra, or algebra courses, and will add focus to a course dedicated to probability. The activities will help students develop the mathematical foundation needed to understand how probability works. There are two sets of activities: One set (Activities 1–7) focuses on certain mathematical basics that are needed to understand applied examples. The other set (Activities 8–26) provides real-world examples. The first set is important because students will be lost when they see a problem if they do not understand a few basic principles and do not have the ability to do basic calculations. The second set forms the bulk of the book, moving from simple experiments to more challenging applications.

After mastering the activities in this book, students will have tools to help them evaluate the probabilities of events they will encounter, and in the process, they will learn to make better decisions in life.

—Eric T. Olson

teacher's page

1. Probability All Around Us

Event	Nonrandom Property	Random Property
weather	seasonal change, local climate	precipitation, temperature on specific days
car accidents	safe or unsafe driving practices	specific cars or conditions met on the road
class grades	amount of study and preparation	appearance of specific questions on tests
customers at mall	hours open, time of day	specific pattern of customer arrival
state lottery	decisions about games offered, prizes	numbers drawn or winning patterns on tickets

2. The term *random* describes events that have underlying variability. Single random events cannot be predicted absolutely. However, using probability and probability models, it is possible to predict the frequencies of outcomes in a large set of random events.

Extension Activity

Have students spend a day or more observing events with random properties and report back to class.

Name ______________________________ Date ______________________

1. Probability All Around Us

After your third softball game in a row is rained out, you are talking with your teammates. One of them says, "Some things just seem to happen. We scheduled these games months ago, but who could have predicted so much rain?"

You ask, "I wonder if this is what my math teacher means by 'random events'?"

Then you all start thinking about how many situations in daily life seem to happen randomly.

1. The table below lists five common events that have both random and nonrandom aspects. Explain what is random and what is not random about each. Then come up with five examples of your own.

Event	Nonrandom Property	Random Property
1. weather		
2. car accidents		
3. class grades		
4. customers at mall		
5. state lottery		
6.		
7.		
8.		
9.		
10.		

2. Describe what is meant by *random event.* Explain the relationship between probability and randomness.

7. Scrambled Word Puzzles

or trying to solve the puzzle first, to narrow down the number of possible arrangements of the letters.

Extension Activity

Have students search the Internet for sites concerning scrambled word puzzles. Some such sites have interactive programs that supply answers for scrambled words. Ask students to speculate on how such programs might work.

Name ______________________________ Date ____________________

7. Scrambled Word Puzzles

Many daily newspapers feature a puzzle in which four or five words are scrambled. You figure out what the words are, then use letters found in certain positions of these words to decode a joke or clever phrase. Use your knowledge of probability to solve this type of puzzle.

1. How many ways is it possible to arrange the 4 letters *A, P, S,* and *T*? Write all the possibilities below. How many of the possibilities are actual words?

   number of possible arrangements ________

   number of actual words ________

2. How many ways is it possible to arrange

   a. 5 distinct letters?

   b. 6 distinct letters?

   c. the letters in the word *GEESE*?

   d. two groups of 3 distinct letters?

3. Complete the puzzle below. Describe any strategies that you used to unscramble the words.

   Probability Puzzler
   Unscramble these four words, putting one letter in each square, to form four words related to probability.

   N A R M O D

   ☐ ☐ ■ ☐ ■ ☐

   S P E S M L A

   ■ ☐ ☐ ☐ ☐ ■ ■

   D C E E U D

   ■ ☐ ■ ☐ ☐ ☐

   C I T R D E P

   ☐ ☐ ☐ ■ ☐ ☐ ☐

   Now arrange the letters in the gray boxes above to answer the following riddle.

   What did Luis find when he turned to the back of his probability textbook?

   ☐ ☐ ☐ ☐ AND ☐ ☐ ☐ ☐

teacher's page

8. Trials: Single Coin Toss

Context

sports

Math Topic

probability distributions

Overview

There is no better way to understand probability than to do simple experiments. Perhaps the easiest way to generate random events is to toss a coin.

Use Activity 9 if you wish to study the binomial model on which these trials are based.

Objectives

Students will be able to:

- conduct and record the results of experimental trials
- make a graph to analyze the results

Materials

- one copy of the Activity 8 handout for each pair of students
- one coin for each pair of students
- graph paper
- calculator

Teaching Notes

Students should pair off. Within each pair, one student will toss or spin the coin to produce good random trials. The other will record the results. Students should switch jobs halfway through the trials. It is recommended that the person tossing the coin catch it rather than let it bounce on the floor.

Students will record 20 trials of 10 tosses each. The exact sequence of heads and tails in each trial should be written on the sheet. For each trial, the students should count and record the number of times the outcome was heads.

Students can then complete a histogram showing the frequency with which heads occurred 0 times, 1 time, 2 times, and so forth, up to 10 times in the trials.

If you decide to do Activity 9, make sure that students save their data from this activity.

Answers

1–2. Results will vary. The theoretical distribution of probability is

0H, $1/2^{10}$

1H, $10/2^{10}$

2H, $44/2^{10}$

3H, $117/2^{10}$

4H, $205/2^{10}$

(continued)

teacher's page

8. Trials: Single Coin Toss

5H, $^{246}/_{2^{10}}$

6H, $^{205}/_{2^{10}}$

7H, $^{117}/_{2^{10}}$

8H, $^{44}/_{2^{10}}$

9H, $^{10}/_{2^{10}}$

10H, $^{1}/_{2^{10}}$

For 20 iterations of the 10 flips, occurrences of 0H, 1H, 9H, and 10H will be rare, with total probability under 5% that any of these will happen. However, many students will experience one or perhaps two of these rare events within their 20 iterations. The rest of the theoretical distribution is 2H, 1 occurrence; 3H, 2 occurrences; 4H, 4 occurrences; 5H, 5 occurrences; 6H, 4 occurrences; 7H, 2 occurrences; 8H, 1 occurrence.

Extension Activity

Have students recreate this activity with a die, investigating the probability of any one of the 6 sides being rolled. One partner will roll the die, and the other will record the results. They should record 20 trials of 10 rolls each. After all the data are collected, ask students to make a histogram showing the frequency with which their selected number (1–6) occurred.

Name ______________________________ Date ______________________________

8. Trials: Single Coin Toss

You know there is a 50–50 chance that your team will win the coin toss at the beginning of a football game. But what is your team's probability of coming out ahead (or behind) in winning the toss during the course of a 10-game season? Find out in this simulation.

1. One partner will toss or spin a coin to produce good random trials. The other will record the results on this sheet. Switch jobs halfway through the trials. It is recommended that the person tossing the coin catch it rather than let it bounce on the floor. Record 20 trials of 10 tosses each. The exact sequence of heads and tails should be written in the table under "Outcomes." For each trial, record the number of times the outcome was heads out of the 10 tosses.

Trial	No. of Heads	Outcomes	Trial	No. of Heads	Outcomes
1			11		
2			12		
3			13		
4			14		
5			15		
6			16		
7			17		
8			18		
9			19		
10			20		

2. After all data are obtained, make a histogram on a sheet of graph paper showing the frequency with which heads occurred 0 times, 1 time, 2 times, and so forth, up to 10 times in the trials.

9. Analysis: Single Coin Toss

Context

simple experiment

Math Topic

binomial model

Overview

In this activity, students use their data from Activity 8 to study the binomial model. Any independent trials built on events that have two possible outcomes, success or failure, fall under the binomial model. In this activity, students study the single coin toss, where success is defined as an outcome of heads and failure as an outcome of tails.

Objectives

Students will be able to:

- understand the binomial distribution
- compare experiment and theory for 10 coin tosses

Materials

- one copy of the Activity 9 handout for each student
- data from Activity 8
- calculator

Teaching Notes

To determine the likelihood of getting a certain number of heads out of 10 tosses of a coin, it is necessary to understand a mathematical model of this situation: the binomial model. This activity lets students calculate the binomial probabilities for their data and compare the calculated probabilities to their actual results.

The term *binomial* follows from the expression $(p + q)^n = 1$. When this expression is expanded into all of its binomial terms, the terms, each of which represents a certain number of successes and failures, all add up to 1. The general expression for an individual binomial probability (P) is:

$$P_{n,m} = \binom{n}{m} p^m q^{(n - m)}$$

In this expression, n is the number of trials, m is the number of trials with successes, p is the probability of success, and q is the probability of failure $(1 - p)$.

It may be easier to teach students how to use the binomial model for a specific case. Say you want to know, before tossing a coin 10 times, the probability of getting 7 heads. First, you need to determine how many ways there are to "choose" 7 events (i.e., get heads 7 times) out of 10 events (i.e., 10 tosses), because any combination of 7 events being heads would result in a total of 7. From our discussion on combinations, we know there are $(10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4)/7!$, or 120, such combinations.

(continued)

Name ______________________________ Date ____________________

9. Analysis: Single Coin Toss

1. Complete the table showing the probability of each of the possible outcomes of a series of 10 coin tosses. Use the data you collected in Activity 8.

   Number of trials from Activity 8: _____

No. of heads	No. of combinations	Probability of individual outcome	Total probability	No. of heads from actual trials	Percent of heads from actual trials
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					

2. What have you learned about the meaning of a mathematical probability of $\frac{1}{2}$ from observing and recording many actual events with this probability?